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SOME IDEAS ON THE STUDY OF GEOMETRY.

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In the preparation of this paper I have followed certain suggestions on the study of Geometry contained in the Report of the Committee of Fifteen on a Geometry Syllabus. The introduction to this report gives an historical account of various attempts to reform the teaching of geometry. This section should prove, therefore, not only interesting but extremely valuable to students and teachers of mathematics. From it we gleam some very noteworthy facts in regard to the progress that has been made in the attempts to place the study of geometry on a sane and satisfactory footing in the school curriculum.

Beginning as a practical subject, an outgrowth of primitive practice in mensuration and physical research, geometry soon proved to be a very fertile field for the study of logic and for the use of formal processes of reasoning. Under the influence of Pythagoras, the application of geometry to practical affairs was at first extended, since he associated it very closely with arithmetic. But the Platonic school despised the science of numbers; they considered the application of geometry to arithmetic and surveying as degrading and vulgar. This school, therefore, emphasized the logical aspect of the subject and did much to increase the rigor and to improve the methods employed in its demonstrations. When Plato inscribed over the entrance to his Academy the words, "Let no one who is not acquainted with

geometry enter here," he had reference, I am sure, to training in the formal processes of reasoning which this subject so beautifully exemplifies and which is a necessary preparation for the study of higher philosophy, rather than to knowledge of the principles of geometry, as geometry.

The great name, of course, in the early history of the subject is that of Euclid, the greatest systematizer of his age, perhaps of any age, who collected and organized all that was then known of geometry into an almost perfect piece of logic. A most remarkable fact of mathematical history is that this body of knowledge as devised by him for adult students over two thousand years ago should have persisted almost unchanged through all these centuries, and be taught at the present time to boys and girls of fifteen or sixeen years of age, as part of their preparation for the general activities of life. It seems to me that we should remember that geometry was originally designed for adults, when we find, as we too often do, that a large number of our students fail to make satisfactory progress in the study of geometry as generally taught. For we require of our children that they be able to give rigorous demonstrations and discover original proofs of abstract propositions, when the keen-minded, speculative Greeks, of mature age, found the same tasks laborious, and the royal Ptolemy asked for "an easier road to geometry."

Another point of interest to me as a teacher of geometry is the attitude which leading mathematicians have, from time to time, taken toward the "Elements" of Euclid. Cajori describes it as the "ever-recurring 'see-saw' between the strictly logical presentation as represented by Euclid, and the more intuitive form which makes greater use of concrete material." It seems to me that it is this latter idea which has gained greatest favor and is to-day being generally accepted.

In France, the attitude almost from the beginning was against Euclid, and after a century or more of criticism and experiment, a great text on "Elementary Geometry" was written by Legendre. Its style was more attractive than that of Euclid, and it placed more emphasis on intuition and less on logic; it contained more matter on solid geometry, and had many points of fusion with arithmetic and trigonometry. This book became the lead-

ing text in France, and had great influence in shaping the early courses in geometry in America. The French have continued to advance in the direction just indicated, and in recent texts and official courses of study, stress is laid upon the use of motion in geometry and upon practice in geometrical drawing; the practical is emphasized wherever possible, and intuition is more fully recognized than ever before; the division of the subject into plane and solid is not adhered to, and the laboratory method —an English idea—is being experimented with. If there is anything in the experience of the French by which we could profit in our work, it is, I should say, in requiring less rigor and in giving freer play to intuition. When a pupil says "I can see that is true without proving it," he should more frequently be given credit for his insight rather than be confused by hair-splitting logical distinctions. I am fully convinced that we often cause a proposition to become hazy in the mind of a pupil and make the whole subject difficult and vague to him by requiring a formal proof when a fact is fully appreciated without it. Other points which we might give more attention to are greater attention to the practical, to the fusion of certain parts of plane and solid geometry, and to the application of algebra and trigonometry when it can be conveniently and profitably done.

In Germany, during the nineteenth century, the influence of Pestalozzi and Herbart made itself felt and gave rise to the genetic, or so-called "heuristic" method of presenting the subject. This is considered the natural method of presentation, since it gives the pupil freedom to express himself in his own way and allows him to discover for himself geometrical truths. The movement toward making geometric instruction genetic is supported by Felix Klein, president of the International Commission on the Teaching of Mathematics, and one of the greatest of living mathematicians, and it is being generally adopted in the schools of Germany. Some study of this method, its aims and its results, is not out of place, therefore, in a paper of this kind.

Herbart gives us the principle that "the pupil must know from the beginning what is aimed at, if he is to employ his whole energy in the effort of learning." In many of the demonstrations of the present-day texts, the opposite principle seems to

have been employed, for the pupil is carefully kept in the dark as to the purpose of the method of procedure until the conclusion is suddenly reached. A fine example of this artificial method may be seen in the usual proof of the theorem concerning the square of the side opposite an acute angle of a triangle. When teaching this theorem, I always find it necessary to invert the order of the steps, beginning with the square of the opposite side, and make such substitutions as we find it possible to make from the relations existing between the various lines until we derive the desired result. Through this procedure I find the pupil able to discover the purpose of each step, and I believe it appeals to him as a reasonable process. A pupil should also, it seems to me, be let into the secret as to the reason for drawing an auxiliary line in a certain way. Does not this hidden plan, so frequently employed, encourage, yes even compel, a pupil to memorize his lesson? Is it not better pedagogy and will the results not be at least as good if with the aid of leading questions we permit the pupil to discover the theorem as well as the plan of its proof for himself? To use a single illustration, when we come to the theorem concerning the lateral area of a prism, why not have the pupil see or handle a prism, discuss with him the area of each of its faces, and let him draw his own conclusion? He will thus have formulated the theorem for himself, determined the steps in the proof of it, and having done this once, he should be able to do it again. In this way the necessity of his memorizing a proof will have been obviated. He will also have discovered what previous theorems are necessary in his proof, and will consequently appreciate more fully the relation between various parts of the subject than would be possible by the more formal method. If the teacher will take part of the period to study with the class the next day's lesson, I believe the subject will be made more interesting to his pupils, and they in turn will get a more intelligent view of geometry as a whole, and derive at the same time greater power of attack in original work. One might add that the habit of requiring pupils to give in advance, and perhaps informally, a definite plan of proof will also aid in securing this result. When a pupil draws an auxiliary line, he should be able to tell what he plans to accomplish with it; if two lines are to be proved equal, he

should be able to give his plan of attack. And so in the demonstration of many theorems, he should be required to outline the steps of the proof before he begins to give it.

Another movement of considerable significance also began in Germany—that of a preliminary course in observational and constructive work in geometry. This idea is meeting with much favor in our schools, and in many places such courses are being introduced in the upper grades. The Committee of Fifteen, in its Report, recommends such a course, and there are many reasons why it should be generally adopted. Of course, this work should not be demonstrational, but based entirely upon observation, experiment, and measurement. Children are interested in work of this nature, and even in the early grades the desire to handle and construct geometrical forms is strong. Such a course may be easily carried on in connection with courses in manual training; but even elsewhere work in the construction of perpendiculars, the bisecting of lines and angles, the making of patterns containing geometrical figures, paper folding, and the use of ruler and compasses, can be profitably undertaken. The mensuration of simple forms, such as the square, cube, triangle and rectangle, may be studied quite early. In this way the pupil soon becomes familiar with the more obvious properties of various figures. Such courses would certainly enrich the work of the upper grades without adding much weight, and the knowledge thus obtained would not only be valuable for the later courses in geometry, but would add much to the preparation for any work that might follow, in school or out.

If, however, some such work is not done in the grades, it seems to me absolutely essential that enough time be taken from the regular course in geometry to acquaint the student first with the subject matter involved in the course. He should become familiar with the general facts of geometry, the nature of geometric figures, and the use of instruments for geometric constructions, before beginning work in formal demonstration. Very frequently, I believe, students—and some who are not dull students—go through the course in geometry without getting any clear notion of what an angle, or a perpendicular, really is. Clear-cut concepts of the elementary geometric forms in the minds of the pupils before formal geometry is begun will obviate

such difficulties. And concrete work in the early part of the course followed by the gradual introduction of formal logic will aid materially in vitalizing the work in geometry.

What is known as the "Perry Movement" has done much to awaken interest, in America as well as in England, in the effort to make the teaching of geometry more efficient. This reform was begun as a reaction against the extreme rigor of treatment required in English schools, and their absurd system of examinations which required that only one order of theorems be accepted, and that a large part of the geometry be memorized, even to the lettering of the figures. It placed great emphasis upon the practical applications of the subject and its utilitarian value, though it possibly went to extremes in this respect. But we owe to the Perry Reform much of the present tendency in our teaching toward closer correlation between geometry and real life; we are coming to believe now that geometry should be applied to real problems; that is, to problems taken from the pupil's actual experience or from those fields in which he is likely to work later, instead of problems that were interesting to the Greeks, or to the students of the Middle Ages, but which have ceased to appeal in the least to the children of the present. Such problems should be gathered from the pupil's every-day activities, out of situations actually encountered by him, and they should be to some extent, at least, actually formulated by him. The study of anything that cannot be put to use in some way in the actual life of an individual is really of slight educational value. To be educative, study must have motive and interest; so if the problems of algebra, geometry, arithemtic, do not appeal to the pupil as having some connection with living experience, he will have little interest in their solution.

There are many advantages, it seems to me, that come from a skillful use of such new material in our courses in mathematics, though there may be also certain disadvantages. In the gathering of real roblems, many artificial ones may appear, problems which are made to fit a certain situation but which have no real existence in actual life; too many problems of various kinds may be admitted, thus bringing confusion and perhaps obscuring the principles to be taught; again, the interest of the class may be dissipated in too much of this work by one's making it an end

in itself, and thus losing sight of the fact that a knowledge of geometry is, after all, the real goal to be sought. On the other hand, the skillful teacher will make such use of these problems as to hold the interest and enthusiasm of his pupils while he is teaching them the principles of pure mathematics. The application of geometry to many such problems will impress upon a student the fact that his mathematics can be used, and will give him knowledge and judgment of how to use it. And the practice obtained in this way will make his algebra and geometry tools which can be used when needed, and used with intelligence and efficiency. But not the least advantage of such concrete, real problems consists in the fact that they rarely "come out even," and the pupil is consequently compelled to use his knowledge of decimals, of common fractions, short methods, and methods of checking results. He will discover that results in actual practice are rarely absolutely correct but only approximate, and he must exercise his judgment as to the degree of approximation necessary in the given problem. And I shall add this advantage, that if we teach the subject in such a way that it gives contact with real, serious, everyday life, rather than spend all the time in the study of pure theory, the study of geometry will furnish increased earning power to the pupil when he leaves school, and enable him to get a better position than he otherwise could.

In the syllabus proper, submitted by the Committee of Fifteen, certain suggestions are made involving change in the manner of treatment and the elimination of some of the theorems. We recall the fact that geometry was originally devised as a study for adult students; in early modern times it was taught only in colleges and universities; and even but a century ago, a graduate of Harvard College, in discussing the course, said, "the sophomore year gave us Euclid to test our strength." If we now teach this subject four years earlier in the course than was done so recently, and to pupils at least five or six years younger, we certainly should teach it differently. The attempt to simplify the course and confine the theoretical side to essential principles is a move in the right direction. Some theorems, including those in incommensurables, require a rigorous demonstration of such nature that their real significance cannot be fully grasped by high school students. The truth of many other

theorems is obvious, and such need only an informal proof. By placing less emphasis upon these theorems, the teacher will be able to shorten the course at certain points, and supplement elsewhere with new material. This will vitalize the work and make it appeal to the interests of the average pupil, especially those who are not inclined toward abstract reasoning. We should, of course, not cease to be logical in the work of geometry, but we should cease, it seems to me, to teach the subject as pure logic. Geometry may have served well as a basis for the study of logic when taught to adult students, men of maturity of mind. But such conditions no longer exist. Should we not now, rather, put less emphasis on dead formalism and teach more earnestly the living facts of geometry, as geometry, as the study of the properties, construction and measurement of geometrical figures, and the application of these principles to real situations in life? And if we do this, we could well give more attention to geometry as an organized body of knowledge. In it are the basal theorems, those of great importance around which the whole subject clings. Others are important only because they serve to establish the relation or connection between various parts. Still others are mere corollaries, independent largely of the subject as a whole, and having little or no application in practice. The subject of geometry has been taught too much as though all theorems were on a dead level of importance. On the contrary, much stress should be laid upon these few basal theorems, such as the "equal triangle" theorems in plane geometry, and in solid geometry the one beginning, "a line is perpendicular to a plane," or the theorem upon which the mensuration of solids is largely based, "an oblique prism is equivalent to a right prism, etc." Such emphasis will help a student to get a proper view of the subject as a unified whole, an idea of the interdependence of the various parts, especially the relation of the minor theorems to the relatively few major ones. This will also help the student to retain his knowledge of the subject for later use.

I have tried to sketch rapidly and rather briefly some of the leading features in the movement to bring about better teaching in the subject of geometry. I might sum up what I have said in a general way somewhat as follows: So simplify the subject that

it may be of more educational value to, and more easily and fully comprehended by, the class of students to whom it is now generally taught; vitalize the study of geometry by again giving it a foothold in the soil of reality whence it sprang; but retain the systematic, logical treatment of the subject as an essential element in the training for the intellectual needs of life.

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